

Illegal Operations Fun

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It is remarkable that some illegal calculation can give CORRECT answers. What are the correct calculation steps?

WARNING : This article is hazardous to your health in learning Mathematics! ☺

$$1. \quad \frac{266666}{666665} = \frac{26666}{66665} = \frac{2666}{6665} = \frac{266}{665} = \frac{26}{65} = \frac{2}{5}$$

$$\frac{22222}{122221} = \frac{2222}{12221} = \frac{222}{1221} = \frac{22}{121} = \frac{2}{11}$$

$$2. \quad \begin{array}{ll} \sqrt{2025} = 20 + 25 = 45 & \sqrt[3]{512} = 5 + 1 + 2 = 8 \\ \sqrt{3025} = 30 + 25 = 55 & \sqrt[3]{4913} = 4 + 9 + 1 + 3 = 17 \\ \sqrt{9801} = 98 + 01 = 99 & \sqrt[3]{5832} = 5 + 8 + 3 + 2 = 18 \\ \sqrt{88209} = 88 + 209 = 297 & \sqrt[3]{17576} = 1 + 7 + 5 + 7 + 2 = 26 \end{array}$$

$$3. \quad \frac{(a+b)^3 + a^3}{(a+b)^3 + b^3} = \frac{(a+b) + a}{(a+b) + b} = \frac{2a+b}{a+2b}$$

$$\frac{a^2}{(a-1)^2 - 1} = \frac{a}{(a-1) - 1} = \frac{a}{a-2}$$

$$4. \quad \text{(a)} \quad 1\frac{1}{5} \times 6 = 1\frac{1}{5} + 6 = 7\frac{1}{5}, \quad 1\frac{1}{6} \times 7 = 1\frac{1}{6} + 7 = 8\frac{1}{6}, \quad 1\frac{1}{7} \times 8 = 1\frac{1}{7} + 8 = 9\frac{1}{7}$$

So we can deduce that $\left(1 + \frac{1}{a}\right)(a+1) = \left(1 + \frac{1}{a}\right) + (a+1) = a + 2 + \frac{1}{a} \quad !$

$$\text{(b)} \quad 5 \times \frac{5}{6} = 5 - \frac{5}{6} = 4\frac{1}{6}, \quad 6 \times \frac{6}{7} = 6 - \frac{6}{7} = 5\frac{1}{7}, \quad 7 \times \frac{7}{8} = 7 - \frac{7}{8} = 6\frac{1}{8}$$

So we can deduce that $a\left(\frac{a}{a+1}\right) = a - \frac{a}{a+1} = a - 1 + \frac{1}{a+1}$

$$\text{(c)} \quad 4\frac{1}{6} \div \frac{5}{6} = 4\frac{1}{6} + \frac{5}{6} = 5, \quad 5\frac{1}{7} \div \frac{6}{7} = 5\frac{1}{7} + \frac{6}{7} = 6, \quad 6\frac{1}{8} \div \frac{7}{8} = 6\frac{1}{8} + \frac{7}{8} = 7$$

So we can deduce that $\left(a + \frac{1}{a+2}\right) \div \left(\frac{a+1}{a+2}\right) = \left(a + \frac{1}{a+2}\right) + \left(\frac{a+1}{a+2}\right) = a + 1$

$$\text{(d)} \quad 7\frac{1}{5} \div 6 = 7\frac{1}{5} - 6 = 1\frac{1}{5}, \quad 8\frac{1}{6} \div 7 = 8\frac{1}{6} - 7 = 1\frac{1}{6}, \quad 9\frac{1}{7} \div 8 = 9\frac{1}{7} - 8 = 1\frac{1}{7}$$

So we can deduce that $\left\{(a+2) + \frac{1}{a}\right\} \div (a+1) = \left\{(a+2) + \frac{1}{a}\right\} - (a+1) = 1 + \frac{1}{a}$

$$5. \quad (5 - 3x)(7 - 2x) = (11 - 6x)(3 - x)$$

$$5 - 3x + 7 - 2x = 11 - 6x + 3 - x$$

$$\therefore x = 1$$

6. $4^{\frac{2}{3}} \times 2^{-\frac{1}{3}} = (4 \times 2)^{\frac{2-1}{3}} = 8^{\frac{1}{3}} = 2$

7. $\log\left(\frac{4}{3} + 4\right) = \log\frac{4}{3} + \log 4,$ $\log\left(\frac{25}{4} - 5\right) = \log\frac{25}{4} - \log 4$

In general we have:

$$\log\left(\frac{a}{a-1} + a\right) = \log\frac{a}{a-1} + \log a, \quad \log\left(\frac{a^2}{a-1} - a\right) = \log\frac{a^2}{a-1} - \log a, \quad a > 1$$

8. $(x + 3)(2 - x) = 4$

$x + 3 = 4$ or $2 - x = 4$

$\therefore x = 1$ or $x = -2$

So, we can deduce that :

$(x + 1 - a)(1 + b - x) = 1 - a + b$

$x + 1 - a = 1 - a + b$ or $1 + b - x = 1 - a + b$

$\therefore x = b$ or $x = a$!

9. Find the largest angle of triangle with sides 4, 7 and 9 units.

$$\sin C = \frac{9}{7} = 1.2857$$

Since $1 = \sin 90^\circ$ and $0.2857 = \sin 16^\circ 36'$

$\therefore C = 90^\circ + 16^\circ 36' = 106^\circ 36'$

Will it always work with a triangle whose sides are $a, a + b$ and $c + a$ where $a^2 + b^2 = c^2$?

10. $\sin^2 5\theta - \sin^2 3\theta = (\sin 5\theta + \sin 3\theta)(\sin 5\theta - \sin 3\theta) = \sin(5\theta + 3\theta)\sin(5\theta - 3\theta) = \sin 8\theta \sin 2\theta$

Can this be extended to any multiple angle?

11. $\frac{9x^2 - 4y^2}{3x - 2y} = 3x + 2y$,

Reasoning: 9 cancel with 3, x^2 cancel with x giving an x ,
4 cancel with 2, y^2 cancel with y giving a y ,
and the "-" cancel with "-" giving "+".